**Model Vision for Excellence in Mathematics**

**(developed in conjunction with the Success Academy Charter Schools)**

It is a core mission of our school district that every student be prepared to be a confident user of mathematics, a powerful quantitative thinker, and a productive problem solver. This mission can only be achieved within a mathematics program that balances mathematical skills, concepts and applications, with instructional practices that emphasize explanation, justification and number sense. That is, we are committed to, a mathematics program built on teaching and learning that actively engages students in learning experiences that stimulate curiosity, inquiry, joy and deep understanding of the mathematics outlined in the Common Core.

While an effective mathematics program must be *guided* by a clear set of content standards, it must be *grounded* by a shared vision of teaching and learning that is evident in the ongoing interactions among students, teachers and mathematics found in every mathematics lesson.

As advocated by the practices described in the Common Core, the adults must consistently expect and support our students to:

Persevere with solving interesting problems,

Reason abstractly and quantitatively,

Construct viable arguments,

Critique the reasoning of others, and

Model with mathematics.

Thus, teachers must consistently and expertly:

Respond to most student answers with “why?”, “how do you know that?”, or “can you explain your thinking?”;

Craft instruction around powerful tasks that promote reasoning and problem solving

And promote productive struggle;

Elicit, and celebrate alternative approaches to solving mathematics problems conveying to students that we value **understanding** and *not* simply memorizing the right procedure to get the one right answer;

Use and connect multiple representations – for example, models, diagrams, number lines, tables and graphs, as well as symbols – of all mathematical work to support the visualization of skills and concepts;

Take every opportunity to develop number sense by asking for, and justifying, estimates, mental calculations and equivalent forms of numbers;

Create language-rich classrooms that emphasize vocabulary, explanations and solutions in the context of meaningful discourse among students;

Embed the mathematical content students are learning in real world contexts

Devote the last portion of every lesson to formative assessment, for example, an exit slip, to assess student understanding

This vision of teaching and learning in elaborated upon in the following 10 Characteristics of Effective Instruction in Mathematics.

**10 Characteristics of Effective Instruction in Mathematics**

**1. High level intellectual preparation by the adults:** An effective lesson provides multiple opportunities for student learning and must be carefully planned. Prior to teaching a lesson, teachers must:

Have a clear understanding of the mathematical concept, the specific learning expectations for their students and how and where these expectations fit in to the larger unit;

Try out the set of problems, tasks and/or activities that support the specific learning expectations;

Identify a key set of questions and expected student responses that support the problems, tasks and/or activities;

Consider the errors that students are likely to make and misconceptions that students are likely to have, and prepare strategies that address these errors and misconceptions; and

Identify how student mastery will be assessed.

**2. The heart of effective mathematics instruction is an emphasis on thinking, problem solving, and reasoning.** Nearly every survey of business and industry addresses the need for current and prospective workers to be able to reason, question and solve problems. Thus the focus on problem solving as the heart of mathematics gives our students a real world competitive advantage. Effective instruction must consistently include opportunities for students to formulate questions and problems, make hypotheses and conjectures, gather and analyze data and draw and justify conclusions.

**3. Effective mathematics instruction balances and blends conceptual understanding and basic math skills.** Real mathematical literacy is as much about understanding the concept of division, knowing when and why to divide, and being able to interpret the meaning of a remainder as it is about knowing how to use an algorithm to find a quotient. Too often, the focus of instruction is on the one right way to get a single right answer, at the expense of understanding why this is the appropriate mathematics, how it relates to other mathematics, and when such mathematics should be used. For this reason, effective instruction balances a focus on conceptual understanding (e.g., the meaning of area and perimeter and how they are related) with a focus on mathematical skills (e.g., how to find the area and perimeter of plane figures).

**4. Effective mathematics instruction relies on flexibility and multiple representations.** At nearly any moment in nearly any class, we know that students are not necessarily processing the content in the way the teacher is processing the content. For example the teacher may be visualizing “three-quarters” as three out of four slices of a small pizza, while one student “sees” three quarters or 75 cents, another student “sees” three red balloons out of a total of four, and still another student “sees” three-quarters of an inch on a ruler. Effective instruction recognizes that students conceptualize mathematical (and scientific!) concepts in different, but often equally appropriate, ways. Effective instruction requires a deep understanding and deliberate attention to such multiple representations, as well as to accommodating the diverse learning styles within every class.

**5. Effective mathematics instruction uses concrete context and connections to engage students and make content relevant.** Teachers cannot rely on abstractions and rules that are not connected to realistic situations, e.g. F when S = 81 is the function of F = 4 (S - 65) + 10. Teachers must take abstraction and embed them in realistic contexts and situations that bring the mathematics and science to life. In this example, telling students that the speeding fine in a particular state is “$4 for every mile per hour over the 65 mph speed limit plus a $10 handling fee for the Police Department” and asking first for the fine when a driver is going 81 mph and then determining a driver’s speed if they received a find of $102.

**6. Effective mathematics instruction invests time in students’ ability to communicate their reasoning and critique the reasoning of others.** The active, engaged, thinking classroom is a classroom of questions and answers, of inquiry and explanations, of conjectures and justifications, and of written and oral discourse. We know that writing helps to clarify our thinking and that teaching another strengthens our own learning. That is why effective classrooms put vibrant student discourse front and center in the form of explanations, dialogues, arguments and presentations.

**7. Effective mathematics instruction incorporates on-going cumulative review.** One of the most effective strategies for fostering mastery and retention of critical skills is daily, cumulative review at the beginning of every lesson. Teachers do this as part of a daily warm-up that focuses on recent instruction or as a daily “mini-quiz” containing 4 to 6 problems that keep skills sharp, review vocabulary and reinforce conceptual understanding.

**8. Effective mathematics instruction employs technology to enhance learning.** Calculators, computers and scientific instruments are increasingly important tools for supporting learning and making instruction more relevant. Graphing calculators that link symbolic, tabular and graphical representations of functions help develop critical understandings of algebra. Smartboards significantly enhance the impact of such software. But it is not the mere use of technology that enhances learning, any more than it is the use of manipulative materials that “teach.” Rather, it is the deliberate use of technology to support the development of mathematical understanding that impacts learning.

**9. Effective teachers use the results of assessment to drive, and constantly refine instruction.** While tests and quizzes will continue to be important components of assessment, it is how the results of these quizzes and tests are used to assess the impact of teaching, plan re-teaching, prepare individual instruction and design additional diagnosis that translates into better teaching and learning. In addition, effective teachers use tools like observations, student work, and exit slips, to monitor the quality of learning. Finally, the results of a carefully aligned system of quizzes and end of trimester assessments are regularly analyzed to make curricular and instructional modifications.

**10. Plan-teach-reflect-refine: Effective teachers of mathematics are self-reflective.**

Effective teachers replay their instruction, reflecting on what worked and what didn’t, and analyze student responses and work as part of an ongoing cycle. Effective teachers do not do this in isolation. They work collaboratively with colleagues and school leadership to reflect on their teaching and make real time improvements.

**Thoughts on Making Inquiry-based, Conceptually-driven, Sense-making Mathematics the Enacted Norm in Every Mathematics Class Every Day**

Our shared commitment is that every student receives well-planned, well-executed mathematics instruction that consistently reflects our vision of active engagement in thought-provoking tasks, productive discussion about mathematical ideas and common misconceptions, and the individual and collective construction of understanding via problem-solving and inquiry.

This commitment implies that teachers will plan their lessons around **rich tasks** that are supported by **targeted questions** and powerful **lesson debrief discussions.** Such lessons are diametrically opposite the “I show, we practice, you do” model of direct instruction that essentially tells students what to remember and how to get right answers. For example, the “trick” to “invert and multiply” works in the short-term, but does not support mathematics as a sense-making enterprise and does not foster an inherent love of mathematics and its power and beauty.

The problem we face as a community of teachers and leaders is that our vision is **not** widely shared, **not** fully understood or even believed, **not** consistently supported and therefore **not** consistently implemented for all students every day. To begin to address this problem, the chart below briefly summarized what students, teachers and leaders are and are not doing to make inquiry-based, conceptually-driven, sense-making mathematics the enacted norm in every mathematics class.

|  |  |  |
| --- | --- | --- |
| **What students ARE doing:** | **What teachers ARE doing:** | **What leaders ARE doing:** |
| * Actively engaging in solving rich problems that are aligned with the Common Core
* Regularly engaging in productive discourse about their thinking and reasoning
* Grappling with mathematical ideas and making and exploring conjectures about those mathematical ideas
 | * Thoroughly studying the Common Core and other resources to develop an understanding of the key mathematical understandings across a grade, unit, or lesson
* Carefully selecting rich tasks that support reasoning and problem solving
* Anticipating students’ solutions and strategies to each task
* Carefully crafting and asking targeted questions that focus on the key mathematical understandings
* Making frequent use of the “discourse clouds”: Why? Can you explain? Who did it differently? Convince us? How did you picture that?
* Regularly collecting and using formal and informal evidence to assess student understanding of the big mathematical ideas and adjusting their instruction accordingly
 | * Regularly meeting with teachers to help them think through their lesson plans, including clarifying the learning goal, the selection of rich, aligned tasks and the questions to be asked during the lesson
* Co-teaching the lesson in ways that support the teacher and maintain a focus on the learning goals
* Taking notes to support a productive debriefing and action planning session
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| **What students are NOT doing:** | **What teachers are NOT doing:** | **What leaders are NOT doing:** |
| * Solving more than three naked problems from a worksheet without the chance to explain their thinking
* Listening to explanations by the teacher without interruption
* Regurgitating procedures to get answers
 | * Showing students how to solve problems and expecting them to replicate the process solely on the basis of remembering
* Using the phrases “this is the rule” or this is “how you solve this” or “this is what you have to remember” without including reasons, explanations or a focus on WHY
* Allowing students to solve problems without providing any opportunities for feedback

  | * Sitting on the sidelines, not interrupting or participating in the lesson
* Using the co-teaching/coaching process only for evaluation
* Only using co-teaching and coaching, with no opportunities for pre-planning or debriefing
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This vision and these enacted norms can only become consistently implemented when:

* they are deeply understood and accepted as how mathematics is taught within the school community;
* the accompanying vision of effective teaching and learning in our schools is a guiding feature of all collaborative discussions and planning and debriefing conferences;
* all teacher and leader training is driven by how to implement this vision;
* the district assembles and makes available an annotated library of videos that captures elements of this vision.